九十二學年度高級中學數學科能力競賽試題(二)參考解答

(台南一中)

1.【參考解答】: If $n>1$ and $x>0$ then $(1+x)^n>1+nx$. So

$$(1+0.0000001)^{10000000}>1+10000000(0.0000001)=1+1=2$$

So $$(1+0.0000001)^{10000000}>2$$

2.【參考解答】:

(1) $$(\alpha^2-2)(\beta^2-2)=(\alpha^2-2)((1+\frac{1}{\alpha+1})^2-2)$$
$$=(\alpha^2-2)(\frac{2}{\alpha+1}+\frac{1}{(\alpha+1)^2}-1)$$
$$=(\alpha^2-2)(2(\alpha+1)+1-(\alpha+1)^2)(\alpha+1)^{-2}$$
$$=(\alpha+1)^{-2}(\alpha^2-2)(2-\alpha^2)$$
$$<0$$

So 2 is between α^2 and β^2. $\sqrt{2}$ 介於α與β之間。

(2) $$(\alpha^2-2)/(2-\beta^2)=(\alpha^2-2)((1+\frac{1}{\alpha+1})^2-2)$$
$$=(\alpha+1)^2(\alpha^2-2)/(\alpha^2-2)$$
$$>1$$

So β較接近$\sqrt{2}$。

3.【參考解答】:

$$P(z)=200z^5+201z^4+202z^3+203z^2+204z+205$$
$$=200(z^5+z^4+z^3+z^2+z+1)+(z^4+z^3+z^2+z+1)$$
$$+(z^3+z^2+z+1)+(z^2+z+1)+(z+1)+1$$
$$(z-1)P(z)=200(z^6-1)+(z^5-1)+(z^4-1)+(z^3-1)+(z^2-1)+(z-1)$$
$$=(200z^6+z^5+z^4+z^3+z^2+z)-205$$

If $|z|<1$ then

$$\left|200z^6+z^5+z^4+z^3+z^2+z\right|\le 200|z|^6+|z|^5+|z|^4+|z|^3+|z|^2+|z|$$
$$<200+1+1+1+1+1=205$$

So

$$(z-1)P(z)=(200z^6+z^5+z^4+z^3+z^2+z)-205\neq 0$$

Thus $P(z)\neq 0$.

4.【參考解答】: $S_n=n(n+1)/2$，$S_n-1=[n(n+1)/2]-1=(n^2+n-2)/2=(n-2)(n+1)/2$

$$T_n=\prod_{k=2}^{n}\frac{S_k}{S_k-1}=\prod_{k=2}^{n}\frac{k(k+1)}{(k+2)(k-1)}$$

$$=\frac{\prod_{k=2}^{n}k(k+1)}{\prod_{k=2}^{n}(k+2)(k-1)}$$

$$=\frac{\prod_{k=2}^{n}k\prod_{k=3}^{n+1}k}{\prod_{k=1}^{n-1}k\prod_{k=4}^{n+2}k}$$

$$=\frac{3n}{n+2}$$

So

$$S_{2003}=\frac{3(2003)}{2003+2}=\frac{6009}{2005}$$ 。