

# 高雄市高級中學九十一學年度數學競賽複賽試題

## 競試（一）【參考解答】

一、【解】

$$\because a = \sqrt{2} + b \text{ 且 } 2ab + 2\sqrt{2}c^2 + 1 = 0$$

$$\therefore 0 = 2(\sqrt{2} + b)b + 2\sqrt{2}c^2 + 1$$

$$= 2\sqrt{2}b + 2b^2 + 2\sqrt{2}c^2 + 1$$

$$= 2(b^2 + \sqrt{2}b + (\frac{\sqrt{2}}{2})^2) + 2\sqrt{2}c^2$$

$$= 2(b + \frac{\sqrt{2}}{2})^2 + 2\sqrt{2}c^2$$

$$\therefore b + \frac{\sqrt{2}}{2} = 0 \text{ 且 } c = 0$$

$$\therefore b = -\frac{\sqrt{2}}{2}, c = 0, a = \sqrt{2} + b = \frac{\sqrt{2}}{2}$$

$$\therefore a + b + c = 0.$$

二、【解】

$$\because p(x) = [r(x)]^2, p(x) + q(x) = [s(x)]^2, p(x) - q(x) = [t(x)]^2.$$

$$\therefore p(x) = \frac{1}{2}[(p(x) + q(x)) + (p(x) - q(x))],$$

$$\therefore [r(x)]^2 = \frac{1}{2}[s(x)^2 + t(x)^2],$$

$$\therefore [2r(x)]^2 = 2[s(x)]^2 + 2[t(x)]^2 = (s+t)^2 + (s-t)^2,$$

$$\therefore [r(x)]^2 = \left[ \frac{s(x) + t(x)}{2} \right]^2 + \left[ \frac{s(x) - t(x)}{2} \right]^2,$$

$$\text{Let } r(x) = x^2 + 1$$

$$\Rightarrow [r(x)]^2 = x^4 + 2x^2 + 1 = (x^2 - 1)^2 + (2x)^2$$

$$\therefore \begin{cases} \frac{s(x) + t(x)}{2} = x^2 - 1 \\ \frac{s(x) - t(x)}{2} = 2x \end{cases} \Rightarrow \begin{cases} s(x) = x^2 + 2x - 1 \\ t(x) = x^2 - 2x - 1 \end{cases}$$

$$\therefore p(x) = (x^2 + 1)^2$$

$$q(x) = [s(x)]^2 - p(x) = (x^2 + 2x - 1)^2 - (x^2 + 1)^2 = 4x^3 - 4x = 4x(x-1)(x+1).$$

三、【解】

$$(1) f(x) = \frac{\begin{pmatrix} 8 \\ 7 \\ 40 \\ x-1 \end{pmatrix} \begin{pmatrix} 32 \\ x-8 \end{pmatrix}}{\begin{pmatrix} 32 \\ x-8 \end{pmatrix}} \cdot \frac{1}{40-(x-1)} \text{ 或 } \frac{\begin{pmatrix} x \\ 8 \\ 40 \\ 8 \end{pmatrix}}{\begin{pmatrix} 40 \\ 8 \end{pmatrix}} \cdot \frac{8}{x}$$

$$\therefore f(10) = \frac{\begin{pmatrix} 8 \\ 7 \\ 40 \\ 9 \end{pmatrix} \begin{pmatrix} 32 \\ 2 \end{pmatrix}}{\begin{pmatrix} 32 \\ 2 \end{pmatrix}} \cdot \frac{1}{31} \text{ 或 } \frac{\begin{pmatrix} 10 \\ 8 \\ 40 \\ 8 \end{pmatrix}}{\begin{pmatrix} 40 \\ 8 \end{pmatrix}} \cdot \frac{8}{10}.$$

$$(2) \sum_{x=8}^{40} x \cdot f(x) = \sum_{x=8}^{40} x \cdot \frac{\begin{pmatrix} x \\ 8 \\ 40 \\ 8 \end{pmatrix}}{\begin{pmatrix} 40 \\ 8 \end{pmatrix}} \cdot \frac{8}{x} = \frac{8}{\begin{pmatrix} 40 \\ 8 \end{pmatrix}} \sum_{x=8}^{40} \begin{pmatrix} x \\ 8 \end{pmatrix}$$

$$= \frac{8}{\begin{pmatrix} 40 \\ 8 \end{pmatrix}} \begin{pmatrix} 41 \\ 9 \end{pmatrix} = \frac{8 \times 41}{9} = 36.\bar{4}$$

四、【解】

令  $C_b$  先滾動一個角度  $t$ ,

設此時  $C_b$  的圓心為  $O'$

設  $\overrightarrow{OO'}$  與  $x$  軸之正向夾角為  $\theta$ ,

則  $a\theta = bt$ .

$$\therefore t = \frac{a}{b}\theta.$$

$$\Rightarrow t - \theta = \frac{a}{b}\theta - \theta = (\frac{a}{b} - 1)\theta \dots (1)$$

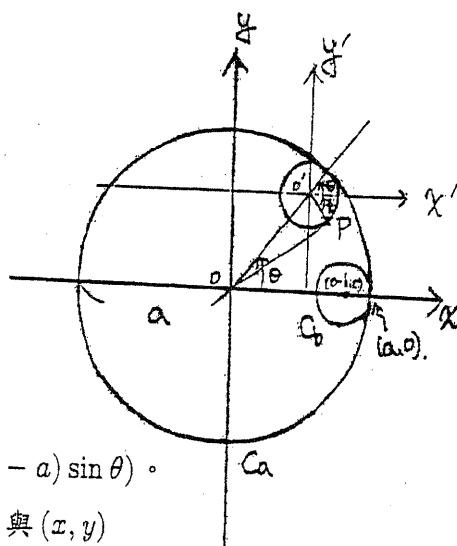
將坐標系平移至以  $O'$  為其原點 (參右圖)

則  $O'$  之於原坐標系之坐標為  $O'((b-a)\cos\theta, (b-a)\sin\theta)$ 。

若此時  $P$  點對於新、舊坐標系之坐標為  $(x', y')$  與  $(x, y)$

$$\begin{aligned} \text{則 } \overrightarrow{O'P} &= (x', y') = (b\cos(1 - \frac{a}{b})\theta, b\sin(1 - \frac{a}{b})\theta) \\ &= (b\cos(\frac{a}{b} - 1)\theta, -b\sin(\frac{a}{b} - 1)\theta) \end{aligned}$$

$$\text{而 } \overrightarrow{OP} = \overrightarrow{OO'} + \overrightarrow{O'P}$$



$$\begin{aligned}\therefore (x, y) &= ((a - b) \cos \theta, (a - b) \sin \theta) + (b \cos(\frac{a}{b} - 1)\theta, -b \sin(\frac{a}{b} - 1)\theta) \\ &= ((a - b) \cos \theta + b \cos(\frac{a}{b} - 1)\theta, (a - b) \sin \theta - b \sin(\frac{a}{b} - 1)\theta).\end{aligned}$$

故動點  $P$  之軌跡方程式為 :  $\begin{cases} x(\theta) = (a - b) \cos \theta + b \cos(\frac{a}{b} - 1)\theta \\ y(\theta) = (a - b) \sin \theta - b \sin(\frac{a}{b} - 1)\theta \end{cases}$