高雄市高級中學九十一學年度數學競賽複賽試題

競試（一）【參考解答】

一、【解】

$\because a=\sqrt{2}+b$ 且 $2ab+2\sqrt{2}c^2+1=0$

$\therefore 0=2(\sqrt{2}+b)b+2\sqrt{2}c^2+1$

$=2\sqrt{2}b+2b^2+2\sqrt{2}c^2+1$

$=2(b^2+\sqrt{2}b+(\frac{\sqrt{2}}{2})^2)+2\sqrt{2}c^2$

$=2(b+\frac{\sqrt{2}}{2})^2+2\sqrt{2}c^2$

$\therefore b+\dfrac{\sqrt{2}}{2}=0$ 且 $c=0$

$\therefore b=-\dfrac{\sqrt{2}}{2},\ c=0,\ a=\sqrt{2}+b=\dfrac{\sqrt{2}}{2}$

$\therefore a+b+c=0.$

二、【解】

$\because p(x)=[r(x)]^2,\ p(x)+q(x)=[s(x)]^2,\ p(x)-q(x)=[t(x)]^2.$

$\therefore p(x)=\dfrac{1}{2}[(p(x)+q(x))+(p(x)-q(x))],$

$\therefore [r(x)]^2=\dfrac{1}{2}[s(x)^2+t(x)^2],$

$\therefore [2r(x)]^2=2[s(x)]^2+2[t(x)]^2=(s+t)^2+(s-t)^2,$

$\therefore [r(x)]^2=\left[\dfrac{s(x)+t(x)}{2}\right]^2+\left[\dfrac{s(x)-t(x)}{2}\right]^2,$

Let $r(x)=x^2+1$

$\Rightarrow [r(x)]^2=x^4+2x^2+1=(x^2-1)^2+(2x)^2$

$$\therefore \begin{cases}\dfrac{s(x)+t(x)}{2}=x^2-1\\ \dfrac{s(x)-t(x)}{2}=2x\end{cases}\Rightarrow\begin{cases}s(x)=x^2+2x-1\\ t(x)=x^2-2x-1\end{cases}$$

$\therefore p(x)=(x^2+1)^2$

$q(x)=[s(x)]^2-p(x)=(x^2+2x-1)^2-(x^2+1)^2=4x^3-4x=4x(x-1)(x+1).$

三、【解】

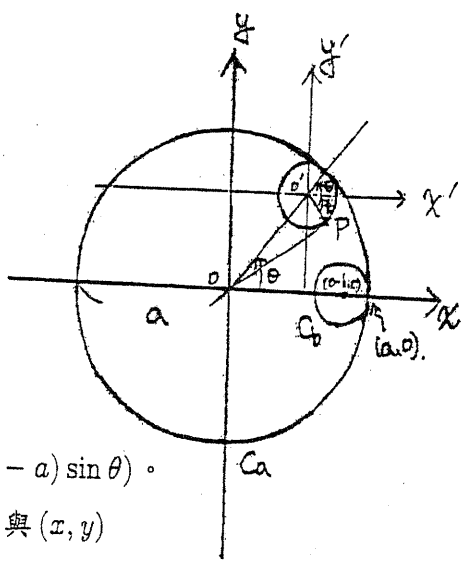
$$(1)\ f(x)=\frac{\binom{8}{7}\binom{32}{x-8}}{\binom{40}{x-1}}\cdot\frac{1}{40-(x-1)}\overset{或}{=}\frac{\binom{x}{8}}{\binom{40}{8}}\cdot\frac{8}{x}$$

$$\therefore f(10)=\frac{\binom{8}{7}\binom{32}{2}}{\binom{40}{9}}\cdot\frac{1}{31}\overset{或}{=}\frac{\binom{10}{8}}{\binom{40}{8}}\cdot\frac{8}{10.}$$

$$(2)\ \sum_{x=8}^{40}x\cdot f(x)=\sum_{x=8}^{40}x\cdot\frac{\binom{x}{8}}{\binom{40}{8}}\cdot\frac{8}{x}=\frac{8}{\binom{40}{8}}\sum_{x=8}^{40}\binom{x}{8}$$

$$=\frac{8}{\binom{40}{8}}\binom{41}{9}=\frac{8\times 41}{9}\doteq 36.\bar{4}$$

四、【解】



令C_b先滾動一個角度t，

設此時C_b的圓心為O'

設$\overrightarrow{OO'}$與x軸之正向夾角為θ，

則$a\theta=bt.$

$\therefore\ t=\frac{a}{b}\theta.$

$\Rightarrow t-\theta=\frac{a}{b}\theta-\theta=(\frac{a}{b}-1)\theta\cdots(1)$

將坐標系平移至以O'為其原點（參右圖）

則O'之於原坐標系之坐標為$O'((b-a)\cos\theta,(b-a)\sin\theta)$。

若此時P點對於新、舊坐標系之坐標為(x',y')與(x,y)

則$\overrightarrow{O'P}=(x',y')=(b\cos(1-\frac{a}{b})\theta,b\sin(1-\frac{a}{b})\theta)$

$\quad=(b\cos(\frac{a}{b}-1)\theta,-b\sin(\frac{a}{b}-1)\theta)$

而$\overrightarrow{OP}=\overrightarrow{OO'}+\overrightarrow{O'P}$

$$\because (x,y) = ((a-b)\cos\theta, (a-b)\sin\theta) + (b\cos(\frac{a}{b}-1)\theta, -b\sin(\frac{a}{b}-1)\theta)$$
$$= ((a-b)\cos\theta + b\cos(\frac{a}{b}-1)\theta, (a-b)\sin\theta - b\sin(\frac{a}{b}-1)\theta).$$

故動點P之軌跡方程式為：
$$\begin{cases} x(\theta) = (a-b)\cos\theta + b\cos(\frac{a}{b}-1)\theta \\ y(\theta) = (a-b)\sin\theta - b\sin(\frac{a}{b}-1)\theta \end{cases}$$