

八十九學年度高級中學數學科能力競試
屏東區試題 (二) 參考解答

1. 參考解答

$$\text{令 } f(x) = (x-b)(x-c)Q_1(x) + 3x - 1 = (x-c)(x-a)Q_2(x) + x + 1$$

$$= (x-a)(x-b)Q_3(x) + 2x + 3 \quad Q_1(x), Q_2(x), Q_3(x) \text{ 爲次數大於 } 2 \text{ 的多項式}$$

$$\therefore f(a) = a + 1 = 2a + 3 \Rightarrow a = -2 \quad \therefore f(-2) = -1$$

$$f(b) = 3b - 1 = 2b + 3 \Rightarrow b = 4 \quad \therefore f(4) = 11$$

$$f(c) = 3c - 1 = c + 1 \Rightarrow c = 1 \quad \therefore f(1) = 2$$

$$\text{令 } f(x) = (x-a)(x-b)(x-c)Q(x) + px^2 + qx + r \quad p, q, r \text{ 爲實數}$$

$Q(x)$ 爲次數大於 1 的多項式

$$\therefore \begin{cases} p + q + r = 2 \dots\dots\dots(1) \\ 4p - 2q + r = -1 \dots\dots\dots(2) \\ 16p + 4q + r = 11 \dots\dots\dots(3) \end{cases}$$

由(2)-(1), (3)-(2)得

$$\begin{cases} 3p - 3q = -3 \\ 12p + 6q = 12 \end{cases} \Rightarrow \begin{cases} p - q = -1 \dots\dots\dots(4) \\ 2p + q = 2 \dots\dots\dots(5) \end{cases}$$

$$\text{由(4)+(5)得 } p = \frac{1}{3}, q = \frac{4}{3} \text{ 則 } r = \frac{1}{3}$$

$$\therefore r(x) = \frac{1}{3}x^2 + \frac{4}{3}x + \frac{1}{3}$$

2. 參考解答

令此最簡分數 $\frac{x}{y}$ 爲且令某數爲 A

$$\text{由題意可知 } \begin{cases} \frac{x+A-7}{y-9} \\ \frac{x-A-1}{y-2} \end{cases} \Rightarrow \begin{cases} 9x - 7y = 9A \dots\dots\dots(1) \\ 2x - y = 2A \dots\dots\dots(2) \end{cases}$$

$$\Rightarrow (1)+(2): 2x = \frac{23}{18}y \Rightarrow \frac{x}{y} = \frac{23}{36} \text{ 且 } A=5$$

3. 參考解答

$(a-1)x^2+(a^2+2)x+(a^2+2a)=0$, $a \in N$ 其解分別如下:

(i) 當 $a=1$ 時 $x=1$

(ii) 當 $a>1$ 時 $x = \frac{(a^2+2) \pm \sqrt{(a^2+2)^2 - 4(a-1)(a^2+2a)}}{2(a-1)}$

$$\frac{-(a^2+2) \pm \sqrt{[(a-1)^2 - 3]^2}}{2(a-1)}$$

(a) 當 $a=2$ 時 $x=2$ 或 4

(b) 當 $a>2$ 時 $x=a$ 或 $1 + \frac{3}{a-1}$ (不為整數)

因此, 當 $a, b \in N, a \neq b$,

$$(a-1)x^2+(a^2+2)x+(a^2+2a)=0 \text{ 與 } (b-1)x^2+(b^2+2)x+(b^2+2b)=0$$

有公共實根 r , 則 $r=4$, 即 $a=2, b=4$ 或 $a=4, b=2$

$$\text{因此 } a^2+b^2=2^2+4^2=32$$

4. 參考解答

$$0^\circ < A+B < 180^\circ$$

$$\therefore \frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2-b^2}{a^2+b^2} \Rightarrow a^2[\sin(A-B)-\sin(A+B)] + b^2[\sin(A-B)-\sin(A+B)] = 0$$

$$\Rightarrow a^2[(\sin A \cos B - \cos A \sin B) - (\sin A \cos B + \cos A \sin B)] +$$

$$b^2[(\sin A \cos B - \cos A \sin B) + (\sin A \cos B - \cos A \sin B)]$$

$$= a^2(-2\sin B \cos A) + b^2(2\sin A \cos B) = 0$$

$$\text{由 } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \text{ 及 } 2\sin A \cos B \cdot a^2 = 2\sin A \cos B \cdot b^2$$

$$\text{可得 } \frac{\sin A}{a} \cos B \cdot b = \frac{\sin B}{b} \cos A \cdot a, \text{ 即 } b \cos B = a \cos A$$

$$\text{由 } \frac{b}{a} = \frac{\sin A}{\sin B} \text{ 及 } b \cos B = a \cos A \text{ 可得 } \sin B \cos B = \sin A \cos A$$

$$\text{即 } \sin 2B = \sin B \cos B = \sin A \cos A = \sin 2A \text{ 則 } 2A = 2B \text{ 或 } 2A = 180^\circ - 2B$$

$$\text{即 } A = B \text{ 或 } A + B = 90^\circ$$

因此為等腰三角形或直角三角形

5. 參考解答

$$2a^2 + \frac{a}{ab-b^2} = 2a^2 + \frac{1}{a-b} + \frac{1}{b} \dots\dots(1)$$

因 $a > b > 0$ 設 $a = b + K, K > 0$

$$(1) \text{式可化為 } 2(b+K)^2 + \frac{1}{K} + \frac{1}{b} \dots\dots(2)$$

由函數 $f(x) = \frac{1}{x}$ 的圖形為凹口向上, 可知

$$\text{當 } b > 0, K > 0 \quad f\left(\frac{b+K}{2}\right) \leq \frac{1}{2}f(b) + \frac{1}{2}f(K)$$

$$\text{即 } \frac{2}{b+K} \leq \frac{1}{2}\left(\frac{1}{b} + \frac{1}{K}\right) \quad \text{即 } \frac{1}{b} + \frac{1}{K} \geq \frac{4}{b+K}$$

$$(2) \text{式可化為 } 2(b+K)^2 + \frac{1}{K} + \frac{1}{b} \geq 2(b+K)^2 + \frac{4}{b+K} \dots\dots(3)$$

$$\therefore 2a^2 + \frac{a}{ab-b^2} = 2(b+K)^2 + \frac{1}{K} + \frac{1}{b} \geq 2(b+K)^2 + \frac{4}{b+K} = 2a^2 + \frac{4}{a} \quad (\text{令 } a = b+K)$$

$$\text{令 } g(x) = 2x^2 + \frac{4}{x}, x > 0, \text{ 得 } g'(x) = 4x - \frac{4}{x^2} \quad \text{令 } g'(x) = 4x - \frac{4}{x^2} = 0 \text{ 得 } x = 1$$

得知 $g''(x) = 4 + \frac{8}{x^3} > 0$ 之極小值為 $g(1) = 6$

$$\therefore 2a^2 + \frac{4}{a} \geq 6 \quad \text{即 } 2a^2 + \frac{a}{ab-b^2} \geq 6$$

又等號出現於 $a = 1, b = \frac{1}{2}$