

八十九學年度高級中學數學科能力競試  
屏東區試題（一）參考解答

1. **解答**

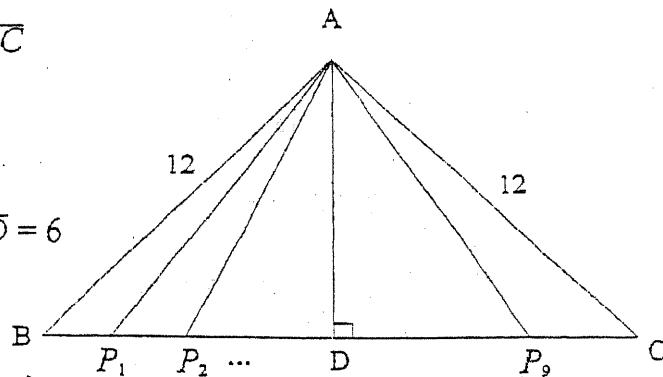
如右圖

則在  $\overline{BC}$  中取一點 D 使得  $\overline{AD} \perp \overline{BC}$

$$\therefore \overline{AB} = \overline{AC}, \angle BAC = \frac{2}{3}\pi, \overline{BC} = 12$$

$$\text{則 } \angle BAD = \angle CAD = \frac{2}{3}\pi \text{ 且 } \overline{BD} = \overline{CD} = 6$$

我們知道  $x_i = \overline{AP_i^2} + \overline{BP_i} \times \overline{CP_i}$



$$\text{則 } \sum_{i=1}^9 x_i = \sum_{i=1}^9 (\overline{AP_i^2} + \overline{BP_i} \times \overline{CP_i}) = \sum_{i=1}^9 \overline{AP_i^2} = \sum_{i=1}^9 \overline{BP_i} \times \overline{CP_i}$$

$$= [\sum_{i=1}^9 (\overline{AD}^2 + \overline{PD}^2)] + \sum_{i=1}^9 \overline{BP_i} \times (12 - \overline{BP_i}) = \sum_{i=1}^9 [12 + (\overline{BD} - \overline{BP_i})^2] + \sum_{i=1}^9 [12\overline{BP_i} - \overline{BP_i^2}]$$

$$= 12 \times 9 + \sum_{i=1}^9 (6 - \overline{BP_i})^2 + 12 \cdot \sum_{i=1}^9 \overline{BP_i} - \sum_{i=1}^9 \overline{BP_i^2}$$

$$= 108 + \sum_{i=1}^9 (36 - 12\overline{BP_i} + \overline{BP_i^2}) + 12 \sum_{i=1}^9 \overline{BP_i} - \sum_{i=1}^9 \overline{BP_i^2}$$

$$= 108 + 324 - 12 \sum_{i=1}^9 \overline{BP_i} + \sum_{i=1}^9 \overline{BP_i^2} + 12 \sum_{i=1}^9 \overline{BP_i} - \sum_{i=1}^9 \overline{BP_i^2}$$

$$= 108 + 324 = 423$$

2. **解答**

$n$  為正整數，拋物線  $y = 2^{2n+1}x^2 - 3 \cdot 2^n x + 1$  與  $x$  軸交於  $P_n, Q_n$  2 點，與  $y$  軸交於  $R_n$

當  $x=0$  時  $y=1 \therefore$  對於所有的  $n, R_n = (0, 1)$

當  $y=0$  時 解方程式  $2^{2n+1}x^2 - 3 \cdot 2^n x + 1 = 0$

$$\Rightarrow x = \frac{3 \cdot 2^n \pm \sqrt{(3 \cdot 2^n)^2 - 4 \cdot (2^{2n+1}) \cdot 1}}{2 \cdot 2^{2n+1}} = \frac{3 \cdot 2^n \pm \sqrt{9 \cdot (2^n)^2 - 8 \cdot (2^n)^2}}{2 \cdot 2^{2n+1}} = \frac{3 \pm 1}{4(2^n)}$$

$$\therefore P_n(\frac{4}{4 \cdot (2^n)}, 0), Q_n(\frac{2}{4 \cdot (2^n)}, 0)$$

令  $a_n$  為  $\triangle P_n Q_n R_n$  的面積 則  $a_n = \frac{2}{4 \cdot (2^n)} \cdot 1 \cdot \frac{1}{2} = \frac{1}{4} \cdot \frac{1}{2^n}$

$$\text{則 } a_1 + a_2 + \dots + a_n + \dots = \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{4} \cdot \frac{1}{2^n} = \frac{1}{4} \sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{4} \cdot 1 = \frac{1}{4}$$

### 3. 答案解説

則(i)<sup>2</sup>-(ii)<sup>2</sup> 得  $(\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = \cos^2\gamma - \sin^2\gamma$

$$\Rightarrow \cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta - \sin^2\alpha - 2\sin\alpha\sin\beta - \sin^2\beta = \cos 2\gamma$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos(\alpha + \beta) + \cos(\alpha - \beta) + \cos(\alpha + \beta) - \cos(\alpha - \beta) = \cos 2\gamma \dots \dots (3)$$

$$\Rightarrow 2\cos(\alpha+\beta)\cos(\alpha-\beta) + \cos(2\alpha) + cso(2\beta) = \cos(2\gamma) \dots\dots (4)$$

由(i)<sup>2</sup>+(ii)<sup>2</sup>得  $\cos(\alpha-\beta)=-\frac{1}{2}$  代入(4)中

得  $\cos(\alpha+\beta)=\cos(2\gamma)$  代入(3)中 得  $\cos(2\alpha)+\cos(2\beta)+\cos(2\gamma)=0$

$$(2) \sin^2\alpha + \sin^2\beta + \sin^2\gamma = (1 - \cos^2\alpha) + (1 - \cos^2\beta) + (1 - \cos^2\gamma)$$

$$= 3 - (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) = 3 - \left( \frac{1+\cos(2\alpha)}{2} + \frac{1+\cos(2\beta)}{2} + \frac{1+\cos(2\gamma)}{2} \right)$$

$$= 3 - \left[ \frac{3}{2} + \frac{1}{2}(\cos(2\alpha) + \cos(2\beta) + \cos(2\gamma)) \right] = 3 - \frac{3}{2} \cdot 0 \quad (\text{by (1)})$$

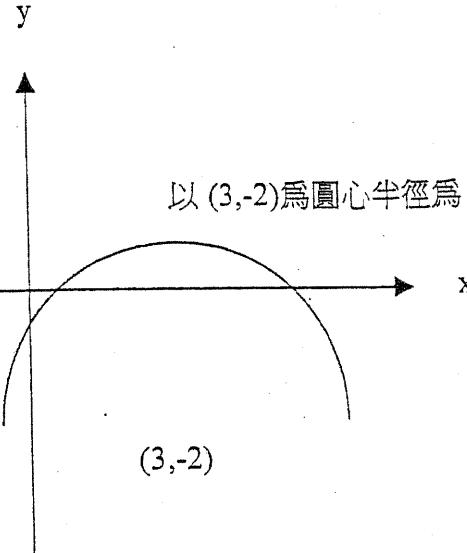
$$= \frac{3}{2}$$

#### 4. 答案解說

$$(1) \quad (y+2)^2 = -x^2 + 6x + 7$$

以  $(3, -2)$  為圓心半徑為 4 的上半圓

$$(x-3)^2 + (y+2)^2 = 16$$



(2)

$$\int_{-1}^7 -2 + \sqrt{-x^2 + 6x + 7} dx$$

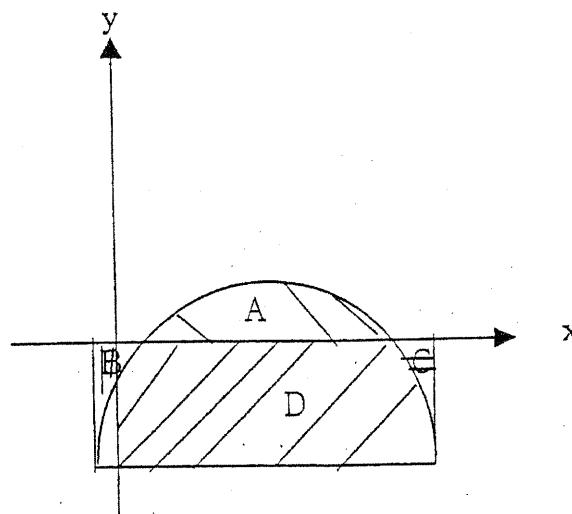
$$= -B + A - C$$

$$= (A + D) - (B + C + D)$$

= 半圓面積 - 長方形面積

$$= 8\pi - 16$$

$$= 8(\pi - 2)$$



## 5. 答

收到資料是錯誤的，但能被視為正確資料的情況出現在 5 條線中有 2 條或是 4 條發生誤差

會發生 2 條誤差的機率為  $\binom{5}{2} \left(\frac{9}{10}\right)^3 \left(\frac{1}{10}\right)^2 = \frac{7290}{10^5}$

會發生 4 條誤差的機率為  $\binom{5}{4} \left(\frac{9}{10}\right)^5 \left(\frac{1}{10}\right)^4 = \frac{45}{10^5}$

其機率為

在錯誤的資料中無法辨識出資料錯誤的機率為  $\frac{7335}{10^5}$

$$\frac{\frac{7335}{10^5}}{1 - \left(\frac{9}{10}\right)^5} = \frac{7335}{40951} \approx 18\%$$