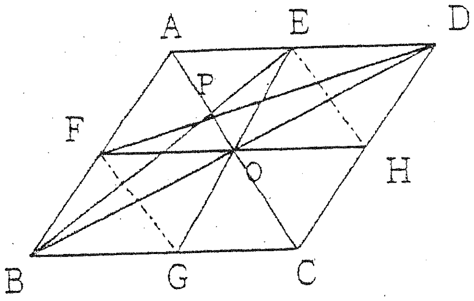
# 教育部八十八學年度高級中學數學科能力競賽決賽

## 筆試試題解答

【筆試問題一參考解答】




在$\triangle$DPA 中,因 $\overline{AF}$ , $\overline{EP}$ , $\overline{DO}$ 交於 B 點,

由西瓦定理知, $\dfrac{\overline{DF}}{\overline{FP}}\cdot\dfrac{\overline{PO}}{\overline{OA}}\cdot\dfrac{\overline{AE}}{\overline{ED}} = 1,$

在$\triangle$DPC 中,由西瓦定理知 $\dfrac{\overline{DF}}{\overline{FP}}\cdot\dfrac{\overline{PO}}{\overline{OC}}\cdot\dfrac{\overline{CH}}{\overline{HD}} = 1,\ \because \overline{AO}=\overline{OC}$

由上面兩式知 $\dfrac{\overline{CH}}{\overline{HD}} = \dfrac{\overline{AE}}{\overline{ED}}$ $\quad\therefore \overline{FH} \,//\, \overline{AC}$

同理：$\overline{FG} \,//\, \overline{AC}$ 故 $\overline{FG} \,//\, \overline{EH}$

【筆試問題二參考解答】

設 $a$ 的正因數依序爲 $a_1 = 1 < a_2 < \cdots < a_r$,

且 $b$ 的正因數依序爲 $b_1 = 1 < b_2 < \cdots < b_s$

$\because a_1 a_2 \cdots a_r = b_1 b_2 \cdots b_s$ 且 $a_k . a_{r-k+1} = a$

$\forall k = 1,2,\cdots r$

$\therefore (a_1 a_r)(a_2 a_{r-1})\cdots(a_r a_1) = (a_1 a_2 \cdots a_r)(a_1 \cdots a_r) = a^r,$

且 $b^s = (b_1 \cdots b_s)(b_1 \cdots b_s)$

$\therefore a^r = b^s$, 設 $a = P_1^{r_1} \cdots P_n^{r_n}, b = P_1^{s_1} \cdots P_n^{s_n}$

$\therefore rr_i = ss_i \quad \forall i = 1,2 \cdots, n$

預證 $r = s$ 否則 $r \neq s$, 設 $r > s, \therefore s_i > r_i, \forall i = 1,2 \cdots, n$

$\therefore r = (r_1 + 1)\cdots(r_n + 1) < (s_1 + 1)\cdots(s_n + 1) = s$ 矛盾

因此 $r = s$

【筆試問題三參考解答】

一.若$f(x)$是質式多項式 ,則$f(x)$即爲所求

二.若$f(x)$可分解

令$f(x) = g_1(x)p_1(x)$其中$g_1(x) = \left(\alpha_i x^i + \alpha_{i-1}x^{i-1} + \cdots + \alpha_0\right)$

$p_1(x) = \left(\beta_j x^j + \beta_{j-1}x^{j-1} + \cdots + \beta_0\right) \qquad i + j = n$

不妨設 $\deg\ g_1(x) \geq \deg\ p_1(x)$

(1).若 $\deg\ g_1(x) < k$時 $\therefore \deg\ p_1(x)$

$\because p \mid a_0$

$\therefore p \mid \alpha_0$或$\beta_0$且$p^2 \nmid a_0$

若$p \mid \alpha_0, p \mid a_0$

$\Rightarrow p \mid \alpha_1, p \mid \alpha_2 \cdots p \mid \alpha_i$(由比較係數模 $p$可得)

$\therefore p$皆整除 $g_1(x)$係數和 $p \nmid a_k$不合

同理若 $p \mid \beta_0$也不合 $\therefore \deg\ g_1(x) \geq k$`

(2). $\deg\ g_1(x) \geq k$

由$p \mid a_0, p^2 \nmid a_0$

設$p \mid \alpha_0 \Rightarrow \quad p \nmid \beta_0$

$\Rightarrow p \mid \alpha_1, p \mid \alpha_2 \cdots p \mid \alpha_{k-1}$

又 $\because p \nmid a_k$

$\therefore p \nmid \alpha_k\beta_0 + \alpha_{k+1}\beta_1 + \cdots \Rightarrow P \nmid \alpha_k\beta_0 \ \therefore p \nmid \alpha_k$

若$g_1(x)$是質式多項式 $\Rightarrow$ 即爲所求

若$g_1(x)$可分爲 $g_2(x)p_2(x)$

$\left[\deg\ g_2(x) \geq \deg\ p_2(x)\right]$

由(1) $\therefore \deg\ g_2(x) \geq k$

再如 (2)討論則有一 $g_m(x)$符合條件

即 $\deg\ g_m(x) \geq k, g_m(x) \mid f(x)$且$g_m(x)$質式多項式 $\therefore$ 命題得證

(3).若由 (2)若$p \nmid \alpha_0; p \mid \beta_0 \Rightarrow$ 一樣若 $j < k$不合 如(1)

若$j \geq k$則就由 $p_1(x)$如(2)討論一樣 $\exists p_q(x)$符合條件